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# Nonlinear Aircraft Loads in Severe Atmospheric Turbulence

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**The problem of predicting loads due to turbulence for aircraft with nonlinear flight-control systems is reviewed, and related recent research in this area is described. Two alternative approaches to the problem, stochastic simulation and worst-case analysis, are compared and problem areas associated with each are identified. It is concluded that the latter method, which involves a search over a specified family of gust patterns to find the pattern that causes maximum load, is appropriate for investigating a limited number of critical cases, but that a preliminary simplified method of estimation is called for by computational considerations as a means of identifying such critical cases. A new genetic algorithm is described and proposed as the most reliable method of performing the search. It is illustrated how this search algorithm can be simplified computationally by introducing a weighting factor that penalizes the more complex gust patterns, a step that is supported by measurements of the amplitudes of gust patterns in severe turbulence.**

## Nomenclature

| | | |
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| $\bar{A}$ | = | measure of response found by power spectral density analysis |
| $a$ | = | empirical constant in Eq. (2) |
| $b$ | = | empirical constant in Eq. (2) |
| $e$ | = | energy-reduction factor |
| $n$ | = | number of component ramp gusts in the discrete gust pattern |
| $p_i$ | = | absolute value of the wavelet coefficient |
| $S_I$ | = | information entropy |
| $U$ | = | energy of the single-wavelet pattern |
| $U_\sigma$ | = | design gust velocity |

## I. Introduction

ACCORDING to a manufacturer's survey [1] of the impact of nonlinear flight-control systems on the prediction of aircraft loads due to turbulence, there is an increasing need to model control-system nonlinearity, to avoid designing control systems that degrade structural performance, and to demonstrate the effectiveness of alleviation systems for aircraft certification. The manufacturer is faced with the conflicting needs to perform a thorough investigation of the effects of such nonlinearity on aircraft loads due to turbulence and at the same time to constrain the potentially rising cost of the load-prediction process. Although a range of analytical tools is currently in existence to investigate this nonlinear problem, according to [1] there is an ambiguity regarding interpretation of the continuous-turbulence (CT) airworthiness requirements [2] that has led to the development of an assortment of methods based on different assumptions and there is a need for the continuing development of nonlinear analysis techniques that provide practical means of compliance with the airworthiness requirements. This paper is intended as a contribution to this process.

Control-system nonlinearity manifests itself in various ways. In [1], the major types of nonlinearity exhibited by the electronic flight-control systems of a typical modern jet transport aircraft are summarized under the following categories:

1) Elemental nonlinearity refers to the existence of nonlinear control-law elements, typically associated with actuator limits and rate limits that can be exceeded when the demand in severe to extreme turbulence becomes sufficiently great.

2) Additive laws are control functions that involve the operation of additional control laws in combination with the normal flight-control laws and are typified by load-alleviation systems. The alleviation function is activated when the excitation due to turbulence becomes sufficiently great and ceases operation when the turbulence subsides.

3) Unidirectional switching is a type of nonlinearity that involves switching between two flight-control functions, for example, from a normal function to a flight-protection function, such as stall protection. Switching to the protection function is triggered by the turbulence, and reversion to the normal flight-control function is only possible with pilot intervention. Consequently, an analysis of the response to turbulence that does not allow for pilot intervention will start in the normal control function but end within the protection function.

Mandatory aircraft limit-load requirements for flight in continuous turbulence [2] are currently formulated in terms of a prescribed power spectral density (PSD) for the turbulence. The load requirement, which follows the design-envelope approach [3], is formulated specifically for linear aircraft response and is expressed in terms of a response factor $\bar{A}$, calculated as a ratio of standard deviations of output and input, multiplied by a specified gust intensity $U_\sigma$ to obtain the design load. The procedure to calculate $\bar{A}$ is generally implemented by means of frequency-domain calculations [3].

When the aircraft response is nonlinear, the preceding formulation of the load requirement in terms of $\bar{A}$ is no longer applicable, and some kind of generalization, compatible with the procedure for linear aircraft, is called for. The standard approach, typified by [4–6], involves stochastic simulation. In this method, the system is excited using synthetic turbulence time histories having Gaussian statistics, based on the von Kármán model [3] for the power spectral density. As described in the preceding references, the procedure for estimating the nonlinear aircraft design gust load is based on the measurement of threshold-exceedance counts for the response.

The application of this stochastic simulation method is faced with two distinct types of problem: one conceptual and one practical. The conceptual problem is that it is inherently tailored to a mission analysis approach rather than a design envelope [3]. Whereas the results obtained are in the form of a statistical distribution, the requirement in the airworthiness regulations [2] takes the form of a design-envelope criterion that calls for the calculation of a single amplitude of response for each load. In the case of a nonlinear aircraft, there is no unique way of calculating this single load from the measured statistical distribution. In practice, approximate methods have been developed [4–6] to estimate the single design load, as required by the design-envelope requirement, from the results of a stochastic simulation.

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The practical problem associated with stochastic simulation is that, even with long simulation run times, there is no guarantee that sufficient information has been collected to adequately define the dynamic response over the critical range of amplitudes in the vicinity of the limit load. The fact that smooth exceedance curves have been obtained can be dangerously misleading, being insufficient to ensure that an adequately representative class of inputs has been generated. For some forms of nonlinearity (for example, those involving some form of switching control system), potentially hazardous conditions may arise only if the input gust pattern lies in a specific limited region of the input space of possible patterns. Furthermore, as usually applied, much of the computer time is spent exciting the system with input patterns, generated at random, for which the shape is far from that of the critical patterns that excite the largest values of response. This is an inefficient use of analysis run time and points to the desirability of developing an alternative procedure, based on the systematic identification of adverse gust-input patterns.

In the context of the assessment of autoland systems in the presence of severe turbulence and wind shear, in which response quantities such as rate of descent at touchdown and position of touchdown along the runway depend in a nonlinear manner on the shear intensity, this view has been expressed in [7], in which the following conclusion was reached: The Monte Carlo (random testing based on stochastic inputs with a prescribed power spectrum) method of the study of hard landings presents a computer-time problem. Because the records of turbulence that cause hard landings are distinctively different from those that do not, it would be possible by means of a computer program to identify those simulated or measured turbulence records that may cause hard landings. If the probability of occurrence of such severe turbulence samples were determined as well, much computer time would be saved. Methods aimed at such systematic identification of adverse gust patterns with known probability are discussed in the following sections.

## II. Procedures Based on Worst-Case Analysis

In 1982, a theoretical basis was provided [8], based on matched-filter theory, for an alternative time-plane based method for the computation of the linear response factor $\bar{A}$, the method of equivalent deterministic variables. This work was developed subsequently in [9]. On the basis of these results, in 1992 an alternative deterministic spectral procedure (DSP) for meeting the PSD load requirement for flight in Gaussian patches of continuous turbulence was formulated [10]. The implementation of this procedure takes the form of a search for a worst-case input that causes maximum load response subject to an energy constraint based on the prescribed power spectrum and gust intensity. In [10], it was pointed out that the energy constraint involved in the DSP makes no reference to system linearity and that, in consequence, the method is applicable equally to aircraft having linear and nonlinear responses. The implementation of the DSP for a nonlinear aircraft was studied in [11]. Over the same period, closely related work on this matched-filter approach was also reported in [12–14]. The analytical basis for the method has also been reviewed in [15].

The problem of treating nonlinear aircraft response to turbulence by means of worst-case analysis has also been studied in the context of the statistical-discrete-gust (SDG) model of turbulence [16–18], which, unlike the DSP, takes into account the non-Gaussian statistics of the more severe gusts. The feasibility of implementing the SDG model when the aircraft response is nonlinear was investigated in a research program described in [19]. The analysis was based on the so-called SDG1 gust model [17,20,21], in which individual discrete gust components are generated by passing pulse-shaped (approximately $e^{-x^2}$) wavelets through appropriate linear shaping filters. The resulting discrete gusts take the form of a ramp followed by a gradual decay.

The implementation of the search for the worst-case response in [19] was structured around a constrained optimization procedure, closely related to the search methods applied to the existing PSD requirement, described earlier [11,14]. However, in contrast with the latter methods, the class of possible input gust patterns to be searched is greatly simplified in the SDG1 model of severe turbulence, being restricted to combinations of only four component wavelets. The implementation described in [19] was applied to configurations involving the combination of a mathematical model of a wide-bodied commercial transport aircraft with four examples of nonlinear load-alleviation systems.

Compared with stochastic simulation, the search for a worst-case input is inherently a more efficient use of computer analysis time, as the inputs generated converge to patterns that cause large amplitudes of response. However, in the case of nonlinear systems, the method is faced with the practical problem that such a directed search may converge to a local maximum of response, rather than to the required global maximum. To address this problem, the search for the critical worst-case input in [19] involved a combination of three separate stages. A purely random Monte Carlo search was selected as the first stage of the process, to supply a starting set for the search procedure. A genetic optimization algorithm [22,23] was chosen for the second stage of the process, and a Nelder–Mead simplex search procedure [24] provided the final stage.

In practice, the application of a such a search procedure to find the global maximum response would be impractical as a means of covering the whole range of load quantities and flight conditions arising in the load certification process and in which the aircraft/system response is nonlinear. A two-level approach is thus called for [1], in which a simplified method is first used to provide a preliminary estimation of the effects of nonlinearity on design loads, and the more comprehensive method is then applied, for a much more restricted range of conditions, to confirm safety in the most critical cases. One such preliminary method, described in [1], is the so-called design stochastic gust approximation. In this method, the worst-case input for a related linearized model of the response is first determined using the criterion specified in the deterministic spectral procedure [10] and then used as input to the nonlinear model. In the design process, this would be used for estimating, to first order, the effects of nonlinearity on design loads for a large number of response quantities/flight conditions [1]. It would also be used to determine a limited number of critical cases to which, in the second level of the process, the more accurate global search method would be applied.

The major novelty introduced in [19] was the use of a genetic algorithm as a key component of the search process. In the following, we describe significant progress made in this area since 1996.

## III. New Genetic Algorithm: Wavelet-Based Differential Evolution

A genetic algorithm formed the core of the search process in the application of the SDG model to nonlinear aircraft response described in [19]. Subsequently, the use of genetic algorithms has become more generally accepted, and in this section, an improved genetic algorithm is described that has been applied to the nonlinear aircraft gust-load problem. As a preliminary, however, some more general comments are made concerning the wider use of genetic algorithms.

A survey of applications of genetic algorithms to aerospace problems was presented in [25]. Following a summary that put genetic algorithms into a broad context, several aerospace applications were reviewed in which this approach has been used successfully. These included the design of a spacecraft attitude controller, the optimization of a homing-missile guidance law, the design of an estimator used to determine satellite orbit parameters, and a solution to the problem of scheduling airport flight arrivals. It is in the light of this widening of general acceptance of the technique that algorithms of genetic type have been developed [26–29] for investigating the performance of nonlinear aircraft systems in severe turbulence.

The particular algorithm to be discussed in the following is differential evolution [30], adapted [27] for the optimization of signals that are represented using wavelets. Differential evolution combines the advantages of breeding and selection, as employed in standard genetic algorithms, with the algebraic operations of addition, subtraction, and averaging. These allow the topography of the cost function to be learned, thus reducing the danger of failing to converge to the required global maximum. The specific steps in the original form of differential evolution [30] may be summarized as follows:

1) Create an initial, randomly chosen, population of trial inputs and evaluate the system response to each one.

2) Breed a new population of inputs according to a prescribed arithmetic formula that combines randomly selected population members with the current best population member (i.e., that which produces the maximum system response).

3) Replace old population members with those newly bred inputs that generate larger system responses. This ensures that the average fitness of the population will increase over generations.

4) Repeat steps 2 and 3 until a convergence test is satisfied.

Differential evolution involves breeding from a population as with previous genetic search algorithms, but the method of breeding is different, making use of addition, subtraction, and division by two, thus requiring the population members to belong to a vector space.

In the adaptation [27] of the method for application to a signal representation using wavelets, the real-valued vectors of classical differential evolution are replaced either with individual wavelets or with wavelet packets, which are used to characterize system inputs. As the wavelet sets do not have a natural vector space structure, it is necessary to substitute appropriate operations for addition, subtraction, and division. A novel feature of the search method described in [27] has been to substitute the vector-space operations with analogous mathematical group operations defined on the space of wavelets, thus exploiting the geometrical structure of the latter to aid the search for worst-case inputs.

As described in [26], this search algorithm is applicable in the context of the current, PSD-based, continuous-turbulence requirement for aircraft loads, using the criterion specified in the DSP described in Sec. II. This calls for a search for the global worst-case input that causes maximum load response subject to a prescribed constraint on the energy of the input. Specifically, this constraint is applied to the energy of a signal comprising a family of pulse wavelets that is passed through a linear gust-shaping filter to generate the input gust pattern.

The performance of the algorithm is demonstrated in [27] by application to a dynamic model of structural loads on a wide-bodied transport aircraft with nonlinear controls, previously used for illustrative purposes in [11], in which full details of the aircraft system and the equations of motion are given. The system output is taken to be aircraft normal acceleration at the center of gravity, known to be closely correlated with wing-root bending moment. The control system (Fig. 1) incorporates a loop in which normal acceleration is fed back through a switch that is initially open, but when the normal acceleration exceeds a prescribed threshold value, the switch becomes permanently closed, allowing uninterrupted feedback. In this application of the algorithm, the population contained 20 members, with each member composed of 10 pulse-shaped ($e^{-x^2}$) wavelets. Ten runs were generated, each consisting of 250 breeding generations. For each of the runs, the final population member with the highest system response was taken and, over all the runs, the greatest of these response amplitudes was chosen to be the search method result.

In [27], it was concluded that wavelet-based differential evolution is a successful implementation of a genetic breeding search method applied to the nonlinear gust-load problem arising from the current PSD-based continuous-turbulence requirement. In particular, it performed advantageously in comparison with the one-dimensional search method proposed by NASA for solving the same problem [14], in terms of both the number of functional evaluations required and the magnitude of the worst-case output found.

## IV. Adaptation of Differential Evolution to Implement SDG Model

The genetic algorithm described in Sec. III was applied in [27] to implement a search for maximum system response under the condition that the family of pulse wavelets that comprises the input to the gust-shaping filters is constrained to have specified energy. Following the criterion specified in the deterministic spectral procedure (Sec. II), it thus provides a method for meeting the existing PSD-based requirement for flight in continuous turbulence.

However, there is experimental evidence [16,17,31] that at high levels of turbulence intensity, atmospheric gust fluctuations are considerably simplified in structure, the more complex patterns being attenuated in relative magnitude. This property is exploited in the SDG model of severe turbulence by means of a modification of the energy constraint used in the DSP. It requires only an elementary modification of the energy constraint applied in [27] to implement a search, based on the SDG gust model, which exploits this simplification in structure. It is proposed that the resulting algorithm has computational advantages for the investigation of design loads on aircraft with nonlinear control systems.

The search routine involving differential evolution [27] can be interpreted as comprising two components. One component controls the evolution of a family of candidate solutions and involves generating an initial family and evolving these by means of the genetic breeding operations. The other component involves the cost function used to measure fitness (expressed in the present context as the magnitude of a structural load) and evaluates this cost function for each candidate solution. The search process involves a closed loop in which information is continually exchanged between these two components.

The adaptation of this routine is as follows. First, for consistency with the SDG1 model of severe turbulence (Sec. II), the von Kármán shaping filter used in [27], which implements the classical one-third scaling law relating gust amplitude to gust gradient distance, is replaced by a modified von Kármán filter [21] that implements the

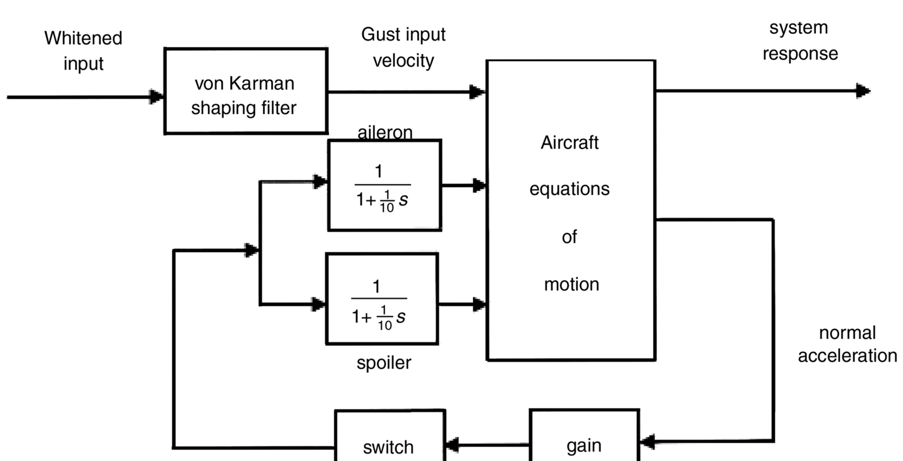


**Fig. 1 Aircraft model with nonlinear flight control system.**

one-sixth scaling law employed in the SDG1 model and that is derived from measurements [16–18] of severe turbulence.

A second change involves the constraint, involving the energy of the associated pattern of wavelets, which restricts the gust patterns to a specified level of probability. In the routine described in [27], applicable to the Gaussian input samples in the PSD model of continuous turbulence, the relevant expression for the energy depends only on the power spectral density of a turbulence velocity component. In [32], this expression is referred to as the spectral-energy function. In the revised routine, to obtain consistency with the SDG1 model, the spectral-energy function is multiplied by the following energy-reduction, or complexity, factors $e(n)$, where complexity is interpreted as the number $n$ of wavelets in the pattern [21]

$$e(1) = 1, \qquad e(2) = 0.77, \qquad e(3) = 0.63, \qquad e(4) = 0.55 \tag{1}$$

For $n > 4$, the value of $e(n)$ is taken to be zero, and thus the maximum number of wavelets required to represent any particular pattern is restricted to four. The consistency of the preceding factors with the measured amplitudes of gust patterns of varying complexity in measured severe turbulence is confirmed in [31]. As the restriction on the number of wavelets greatly reduces the dimensions of the search space, the computer time taken to find the required global extreme value is reduced and confidence that the global extreme value has been found is enhanced.

In illustration of the modified procedure, it is applied here to the mathematical model of an aircraft with nonlinear switching control system (Fig. 1) used previously for illustrative purposes in [11,27]. For the purpose of illustration, the switch was taken to operate at 0.15 $g$. Figures 2a–2d show the shapes of the worst-case pulse-wavelet signal patterns resulting from four independent searches for the input that produces the maximum response in normal acceleration at the center of gravity, each under a constraint on the energy (as in [27]), but in which the energy constraint incorporates the factors given by Eqs. (1). Specifically, the input energy in the cases of two, three, and four wavelets has been taken to be $(0.77)U$, $(0.63)U$, and $(0.55)U$, where $U$ is the energy prescribed in the case of the single wavelet. For the purpose of this illustration, the magnitude of $U$ was chosen arbitrarily such that the operation of the switch had a significant effect on the magnitude of the peak response. In applications to evaluate design loads, the energy $U$ would be calibrated [21] such that the isolated ramp gust generated by passing the single wavelet through the modified von Kármán shaping filter has a specified amplitude at a prescribed gradient distance, typically taken to be 60 ft/s at 430 ft.

The associated maximum-response amplitudes, normalized to the case of the single-wavelet input are 1.00, 1.65, 1.50, and 1.40 and, as the SDG method requires that the maximum value (1.65) of these be taken, the overall tuned input is thus that generated by two pulse wavelets (Fig. 2b). This overall maximum-response amplitude provides the design load predicted by the SDG1 gust model.

Although the constraint on the search space, imposed by limiting the maximum number of wavelets to four, greatly reduces the computational requirement as compared with the method described in Sec. III, there remains scope for further simplification if the required number of independent searches could be reduced from four to one. The feasibility of this further step is demonstrated in the following section.

## V. Introduction of Entropy-Dependent Energy-Reduction Factor

In [32], a generalization of the SDG1 model, referred to as SDG (E), is described in which the four energy-reduction factors $e(n)$ in the SDG1 model [Eqs. (1)] are replaced by a single entropy-based energy-reduction factor:

$$e(S_I) = (1 + aS_I^b)^{-1} \tag{2}$$

where $S_I$ is the information entropy [33] of a wavelet pattern, given by

$$S_I = -\sum_i p_i \log_2(p_i) \tag{3}$$

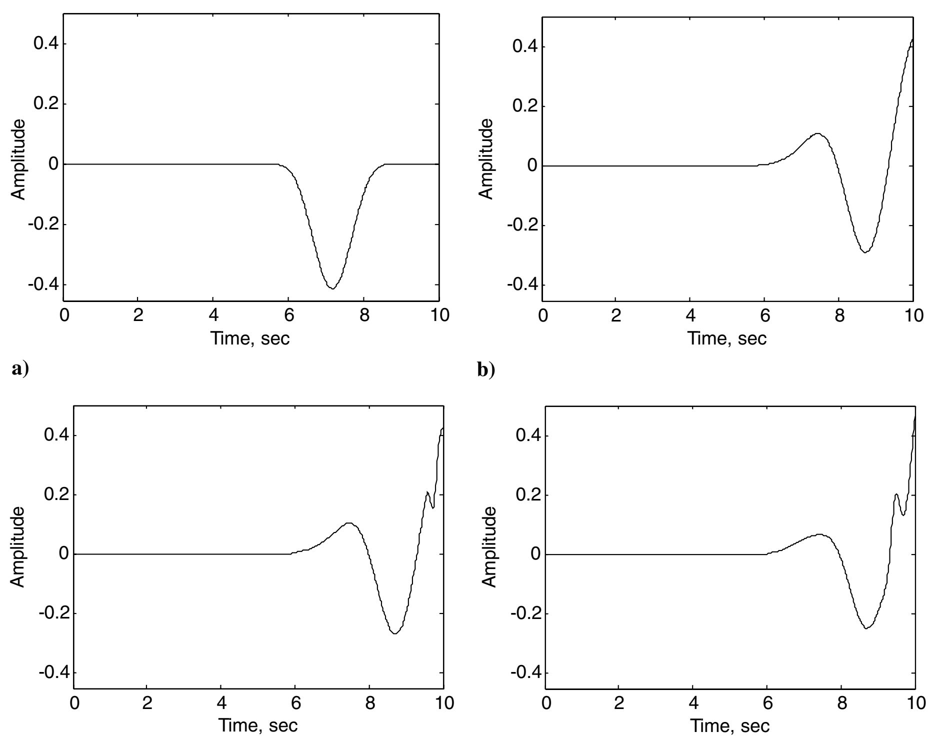


**Fig. 2 Shapes of nonlinear aircraft system tuned inputs. Result of using energy constraints: a) 1 element, energy = $U$, b) 2 elements, energy = (0.77) × $U$, c) 3 elements, energy = (0.63) × $U$, and d) 4 elements, energy = (0.55) × $U$.**

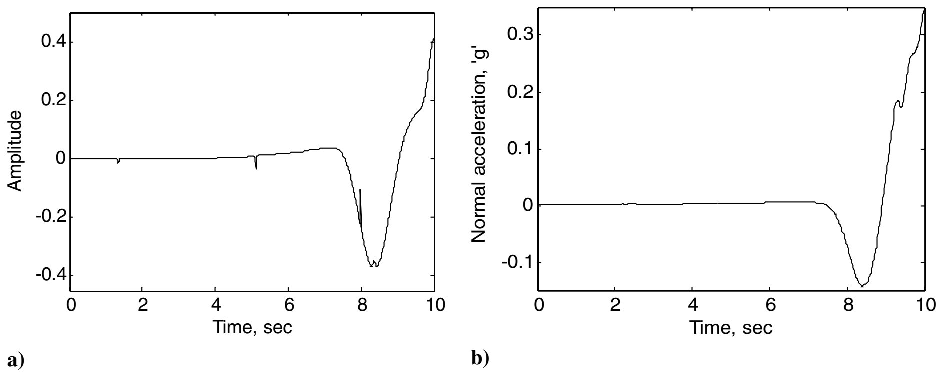


**Fig. 3 Nonlinear aircraft system: a) tuned input and b) tuned response (normal acceleration at the center of gravity). Result of using combined energy/entropy constraint with energy = $e(S_I) \times U$.**

The $p_i$ are absolute values of wavelet coefficients defined in [32], normalized to sum to unity, and $a$ and $b$ are constants determined empirically such that Eq. (2) is compatible with the SDG1 energy-reduction factors $e(n)$. As is the case with the $e(n)$ [Eqs. (1)], the energy-reduction factor $e(S_I)$ is a positive quantity, taking maximum value unity, which decreases as the complexity of the signal, here measured by the entropy $S_I$, increases. If there is just one nonzero wavelet coefficient in Eq. (3), the associated gust pattern comprises just a single ramp gust. Then $S_I = 0$ and $e(S_I)$ [Eq. (2)] takes the value unity.

By matching to values of $e(n)$ derived, for values of $n = 2$ and 4, from severe-turbulence events encountered during routine operational flying, the following approximate values have been derived [32]:

$$a = 0.3, \qquad b = 1.45 \tag{4}$$

In illustration, the method has been applied to the aircraft with a nonlinear switching control system (Fig. 1), already used for illustrative purposes in Sec. IV and in [27]. The wavelet-based search algorithm described in Sec. III, which maximizes the amplitude of the response of a nonlinear system subject to a prescribed constraint on the spectral energy of the input [27], has been modified simply by incorporating the entropy-dependent energy-reduction factor $e(S_I)$, which penalizes the more complex patterns. As in the algorithm described in [27], for this application to a nonlinear system, each input sample comprises 10 pulse-shaped wavelets.

Figure 3a shows the result of applying this SDG(E) algorithm to find the tuned input that maximizes the response in normal acceleration at the center of gravity, and Fig. 3b shows the associated response time history. Subject to the energy/entropy constraint, the associated maximum-response amplitude is 1.75, normalized with respect to the maximum response when the input comprises just a single wavelet having the same specified energy $U$ as in Fig. 2. This value is somewhat greater than the normalized maximum-response amplitude of 1.65 that results (Sec. IV) from the four independent constant-energy searches required in the application of the SDG1 method. The increase in response amplitude is attributed to the more detailed variation in pattern shape resulting from the use of 10 component wavelets in each input sample, in conjunction with the continuous variation of the energy-reduction factor $e(S_I)$. This contrasts with the discontinuous variation of $e(n)$ with the discrete number $n$ of component wavelets, which is limited to a maximum of four.

The associated effects on pattern shape can be seen by contrasting Fig. 3a with Figs. 2b and 2c. It can be seen from Fig. 3a that at least 4 of the 10 wavelets used in the representation are redundant (small spikes). On account of the rapid exponential decay of the function $e(S_I)$ with complexity [Eq. (2)], the pattern shape is dominated by just three or four wavelets, with wavelet components beyond the fourth making only a small contribution to the energy. However, their inclusion in the representation has the advantage of allowing a greater flexibility in detailed pattern shape, at essentially a constant level of probability.

As compared with the algorithm used to implement the deterministic spectral procedure ([27] and Sec. III), in which the constraint is based purely on signal energy, the energy/entropy algorithm converges rapidly to a relatively limited region of the search space, comprising patterns dominated by relatively few wavelet components. For a given amount of computational effort in terms of function evaluations, the neighborhood of the required maximum in response will thus tend to be explored in greater detail. Conversely, if the criterion for comparison is based on a convergence test applied to the response, it is to be expected that the energy/entropy algorithm will require fewer function evaluations. Finally, it may be noted that the application of the energy/entropy constraint, as a means of representing the non-Gaussian structure of severe turbulence, would also be advantageous in other computational applications of the SDG model to the nonlinear optimization problem [28,29,34].

## VI. Conclusions

1) The problem of predicting loads due to turbulence for aircraft with nonlinear flight-control systems has been reviewed, and related recent research in this area has been described.

2) When the response is nonlinear, two alternative approaches to the problem of meeting the airworthiness requirements for flight in turbulence are currently in use: namely, stochastic simulation and worst-case analysis.

3) Stochastic simulation suffers from the disadvantage that it results in a measured statistical distribution, whereas the design-envelope requirement specified in the airworthiness regulations calls for a single amplitude of design load corresponding to a prescribed turbulence intensity. The step leading from the measured distribution to the required single design load is not well defined, and alternative approximate methods for taking this step are in use. In contrast, the method based on worst-case analysis leads unambiguously to the required design load.

4) In the case of stochastic simulation, much of the analysis time is currently spent exciting the system with input patterns, based on Gaussian statistics and generated at random, for which the shape is far from that of the critical pattern(s) that excite the largest values of response. This is an inefficient use of analysis run time and points to the desirability of a procedure aimed at the systematic identification of adverse gust-input patterns.

5) The alternative of worst-case analysis calls for a systematic search over a prescribed class of turbulence input patterns to find the input causing maximum response. Such an approach, based on the criterion specified in the deterministic spectral procedure (DSP), exists in the context of the current continuous-turbulence requirement. When applied to a linear system, the input would converge systematically to the unique pattern that causes the largest amplitudes of response. However, for nonlinear systems, the search may converge to a local, rather than the global, maximum of response. It is

proposed in this paper that genetic algorithms have advantages in dealing with this problem.

6) As a means of covering the whole range of load quantities and flight conditions, the application of a search procedure to find the gust pattern that causes maximum response would be impractical. A two-level combined approach is thus called for in which a simplified method, such as the design stochastic gust approximation [1], is first used to provide a preliminary estimation of the effects of nonlinearity on a wide range of design loads, and the more comprehensive search method is then used to confirm the safety in only the most critical cases.

7) A new genetic search algorithm, wavelet-based differential evolution, has been described. This algorithm combines the advantages of breeding and selection, as employed in standard genetic algorithms, with algebraic operations of addition, subtraction, and averaging. These allow the topography of the cost function to be learned, thus reducing the danger of failing to converge to the required global maximum.

8) It has been described how wavelet-based differential evolution has been applied to a nonlinear aircraft model, using the criterion specified by the DSP for meeting the current PSD requirement, which is based on Gaussian statistics. This criterion incorporates a constraint in which input signals have prescribed energy.

9) It has also been illustrated how, by applying energy-reduction factors, which penalize the more complex gust patterns, wavelet-based differential evolution may be also applied to implement a modified algorithm based on the statistical-discrete-gust (SDG) model. Two forms of the SDG model exist having different, but approximately equivalent, specifications of the energy-reduction factors. One form, the SDG1 model, introduces factors $e(n)$, which depend only upon the number $n$ of elementary components in the gust pattern. The other form, SDG(E), employs energy-reduction factors $e(S_I)$ that depend explicitly upon the information entropy $S_I$ of the gust pattern.

10) By introducing such entropy-dependent weighting factors, the resulting search for the worst-case gust pattern tends to converge rapidly to a region of the search space that excludes the more complex gust patterns, consistent with the measured statistics of severe turbulence. Relative to the unmodified DSP, this has computational advantages in terms of the number of function evaluations required to estimate the desired design load.

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